LE STUDIUM
Loire Valley
Institute for Advanced Studies

LE STUDIUM Multidisciplinary Journal
ISSN 2669-7807
www.lestudium-ias.com

FELLOWSHIP FINAL REPORT

Interdisciplinary analysis of the Leonardo da Vinci's studies on dynamics

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REPORT INFO

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Period of residence in region Centre-Val de Loire: July 2020 - September 2023

Keywords :
Leonardo da Vinci, History of Sciences, Physics, Dynamics, Transdisciplinary approach

ABSTRACT

The activity was mainly addressed to study the scientific production of Leonardo da Vinci by means of an integrated approach that combines exact and human sciences. A huge interest was focused on dynamics, topic of great interest for Leonardo as it emerges in several points of his notebooks where he discussed relevant aspects of this physics branch and developed conceptions that evolved over the time. Within this general goal, the following specific issues have been also taken into account: i) recognition and comparative source analysis of the network of scholars, artisans and texts for a reconstruction of the cultural, scientific and technical framework in which Leonardo elaborated his concepts; ii) impact of the Leonardo's work on the present research fields; iii) impact of the Leonardo's work on the present educational field; iv) evaluation of the Leonardo's work as an innovative tool for the "third culture" promotion. In this framework, the analysis of the Leonardo's studies followed two driving lines: i) to place his findings in his time context and ii) to establish connections with the current cultural approaches.
The activity has been carried out at the Centre d'Etudes Supérieures de la Renaissance (CESR) of Tours. The performed study has a great relevance for the valorisation of the Centre-Val de Loire region where Leonardo spent the last three years of his life.

1- Introduction

The main goal of the performed research activity was to study the scientific production of Leonardo da Vinci, with a special emphasis on dynamics, by means of an integrated approach characterized by a marked interdisciplinary character and based on the analysis of his notes and drawings, of the texts he consulted and of the different contexts where he lived. The employed synergistic approach integrates different disciplines and sources showing that Leonardo did not pursue an approach based on disjunction, reduction and abstraction, i.e. on reality simplification, but an approach which is nowadays reputed essential for investigating complex systems. Although Leonardo was endowed with an articulate, complex and centrifugal intelligence, so intense that he was incapable of focusing on a single subject, in his approach to knowledge, he had a fixed starting point: experience. In turn, he was against those approaches to knowledge not based on direct observation of phenomena, the only acceptable tribunal for knowledge [1,2]. In codex Forster III folio 14r, he wrote: *Wisdom is the daughter of experience.* In Codex Atlanticus 538 r, Leonardo wrote: *where there is no quantity there is no division.* This sentence reflects one milestone of his approach: the phenomenological world is not only a "visible" world but is to be "divisible" and hence "measurable". In this context, he considered the use of mathematics to be fundamental for the study of nature. In Libro di pittura 1v, he wrote:

No human investigation can be called true science, if it does not involve mathematical demonstrations. Leonardo did not limit himself to experience: he also performed experiments, intended as sequence of operations capable to reproduce phenomena under known conditions, conditions not always found in common experience; conditions that can furnish, in some cases, results that are in contrast with common experience. In line with this approach, he investigated the phenomenological world by operational procedures, through informative notes and almost photographic drawings, assimilable to real professional reports, pervaded by a mastery of the notion, a mastery of the representation and of precision.

His scientific activity fell within the gap, that offers a discontinuous vision of science, between an approach to scientific knowledge, based on observational practice followed by a rational discursive practice, and an experimental approach that will characterize the new science, guiding the transition to a new way of investigating Nature.

In a way, to some extent Leonardo marks the transition towards the Galileo Galilei *sensible experiences and certain demonstrations*.

So, Leonardo left us, besides a testimony of passion for truth, also a way to approach knowledge, offering a living example of how truth should be sought, and on how clarity of thought should be pursued.

2- Results and discussion

An example of his approach can be found in the field of "tribology", where, on the basis of real experiments performed with paralepidid blocks arranged on different faces on a plane, Leonardo introduced a law, expressed in natural language rather than in a symbolic form, which was formulated in mathematical terms later, in 1699, by Guillaume Amontons. The law affirming that friction does not depend on the extension of contact surface is counter-intuitive and even paradoxical, being contrary to mere common sense. In particular, in Codex Forster III 72r, Leonardo draws three blocks, pulled by weights, stacked vertically, and arranged horizontally and he writes: *the friction amounts to the double for double weight*. Below the blocks are two isosceles triangles geometrically representing the notion of proportionality used by Leonardo, his *pyramidal law*: when the weight is doubled, the force of friction is doubled. So, he showed that the blocks arranged horizontally and those arranged vertically undergo the same friction. In this folio dedicated to *confrecazione*, i.e. to friction, which is the degrading factor that makes the dynamics of every system limited in time, at the top of the folio one can see a bust of an old lady and a remark in black pencil: *Beautiful mortal house passes and does not extend*.

When it was not possible to reach phenomenological laws, as for the damped pendulum reported in Codex Madrid I f 147r, Leonardo did not abandon a realistic description, using a meticulous representation by images, rich in details, preferring to show than to describe, using drawings to design rather than words to designate. For Leonardo, in fact, drawing was part of an operational language, more direct and immediate than words. He represented the pendulum oscillation in its complexity and without simplification, i.e. without neglecting reality aspects, which was the basis of the Galileo's approach, whose motto's was *difalcare gli impedimenti,* i.e. cutting out hindrances, and hence stretching the true. In the pendulum image, the chosen *sampling time*, which corresponds to the time between one position measurement and the next one, is singular and arouses curiosity. Leonardo chooses it as variable in order to better follow the movement variations: when the pendulum has a higher velocity, i.e. close to the vertical position, the sampling time decreases; vice versa, when the pendulum slows down, i.e. when the pendulum reverses the direction of motion, the sampling time increases. So, he preferred a flexible approach in comparison with the simpler isochrone constant sampling time. With his creative indiscipline, he, without harm, forgot the rules of isochrony in favour of a more effective choice of the sampling time. A witness of how he observed the world, and how he trusted to his inner vision.

Most of the extensive collections of Leonardo's notes and drawings on water flows concern hydraulic engineering problems [1,2]. Leonardo was the first to systematically study the

dynamics of water in a river. For this study he used as motion markers, as measure devices, cork stoppers floating at different distances, and he measured the travel time for each cork. He also designed instruments to measure the differences in levels in rivers. In CA f 1007r, he wrote: *Water runs faster where the channel is narrower, and on leaving the narrow place it widens with fury and damages the near transverse banks, and often changes course in one or the other place*. In Ms A 57v, he wrote: *This proposition is clearly proven because, although the river has equal width and area, and unequal depth, it is necessary that the course of this river, again, have an uneven movement for the reasons given above. Its movement is of this quality. Suppose the figure marked next to it is the river: I say that in m n the water has a faster movement than a b, especially as m n enters a b, which enters it 4 times. Thus, 4 times faster will be the course in m n than in a b and 3 times faster than in c d and 2 times faster than in e f.*

This result is nowadays known as *Leonardo's law* or *law of lift*, lift being defined as the product of the speed for the surface. It is considered a special case of the *continuity equation*, is widely used in physics in the study of the dynamics of liquid systems (hydrodynamics) and is the consequence of the principle of conservation of mass, which Leonardo did not know. The Leonardo's approach also in this case was therefore original and valid in the identification of a quantitative law which is nowadays also called *Castelli's law*, Benedetto Castelli being a Benedictine monk who formalized in his *Della misura dell'acque* this law of hydrodynamics in mathematical terms.

As for the case of friction, Leonardo, by backing solid arguments, transposes with a transdisciplinary analogy the physical behavior of fluids to the flow of people, according to a model that today is often used to quantitatively simulate the mechanisms of pedestrian flow: Leonardo's law is also valid for the collective motion of people. A further example of exchange, fusion, and crossing of disciplines.

Concerning the physics of motion, Leonardo supported the impetus theory, one of whose formulators and supporters had been Jean Buridan. He coped with originality this subject, with countless examples, in terms of *impression duration*. In Codex sur le vol des oiseaux 12r, he wrote: *Every movement tends to be sustained, that is, every moving body continues to move as long as the impression of the power of its motor is preserved in it.* From Leonardo's sentences, it emerges that the movement of a body characterized by a constant impression, or by an infinite memory, will persist indefinitely in its state; vice versa, when the impression of the movement decreases with time, the system stops. In this frame, Leonardo's dynamics include both the dynamics of Aristotle, which corresponds to an infinitely short memory, and that of Newton, which corresponds to an infinitely long memory, and, exceeding expectations, can be inscribed in the modern theory of the linear response.

Leonardo had a constant interest towards the figure and the works of Archimedes of Syracuse, and it is possible to trace correspondences and, for some topics, even signs of continuity. This is the case, for example, in the analysis of the centers of gravity or for some simple machines where the elegant theory, formulated by Archimedes, is followed by Leonardo, through experimental analyses that realistically take into account, also in a quantitative way, the presence of friction. Archimedes and Leonardo interpreted the concept of infinitesimal, a term introduced by Aristotle. In the case of the application of the iterative method of exhaustion, both refer to the abstract *infinitesimal potential*, quantity that can be decreased beyond any limit. Furthermore, it also emerges that Leonardo, thanks to his approach intimately connected with experience, widely uses the concrete concept of the *physical infinitesimal*. More in details, the problem of the circle's squaring was solved by Archimedes, with great elegance, by introducing his spiral. Archimedes shifted the problem of rectifying the circumference to that of tracing the tangent to the spiral, obtaining an exact solution. The same problem was solved by Archimedes by successive approximations based on the determination of the perimeters of inscribed and circumscribed polygons, starting from the hexagon and by successive doubling of the number of sides, i.e. operating with regular

polygons of 6, 12, 24, 48, 96 sides. This approach was known by Leonardo who on Codex Atlanticus f 230r, writes: *The squaring of the circle made by Archimedes is well said and it is well said and badly given. Well said is where he said that the circle is equal to an orthogon made up of the length of the circumference and the semi-diameter of a given circle. And it is badly given where he frames a figure made up of 96 sides, which is missing 96 distinct portions from the 96 sides. And this is in no way to be called squaring the circle; but in truth with these rules, it is impossible to do otherwise.* Then, Leonardo considered a partition of the circle into sixteen circular sectors which, disposed on a straight line, allow to evaluate the approximate area of the circle. The problem of rectifying the circumference and determining the area of the circle is therefore solved, by means of a procedure which includes successive steps. He first breaks down the circular section into sixteen circular sectors; then he proceeds to unroll them on a plane, thus rectifying the circumference, obtaining the profile of a triangular function where the area of the sum of the isosceles triangles approximates that of circular sectors. So, he figures out the problem, with the exact calculation of the sum of the areas of the isosceles triangles; in analysis this is what is referred to as a *partial sum*. Finally, he iteratively extrapolates the straightforward procedure, imagining a resolution of the circle in an ever-increasing number of triangles to obtain an exact result. Leonardo makes explicit reference to the sum of a series of "almost infinite" pyramids which evokes the concept of integral as a limit value of a series. In Codex Madrid II, f. 112r, Leonardo writes about squaring the circle: *The night of Santo Andre I finally found the squaring of the circle; and at the end of the light and of the night and of the paper on which I was writing, was concluded; at the end of the hour.*

But, for Leonardo the phenomenological world is not only a “visible” world but is to be “divisible” and hence “measurable”. And then he introduces the concrete concept of *physical infinitesimal*. An example is found in Codex Leicester f. 6r, where Leonardo writes: *The water, which is not in motion, does not weigh at its bottom, as can be seen from the thin grasses above the bottom among the wavy water, and in the very light mud of the marshes, which is almost of the lightness of water , which, if the water were to rest on top of it, would come to condense and almost petrify: which, showing itself to the contrary, states that the water above its bottom does not weigh, it concludes from this experience.* Here Leonardo takes into consideration a liquid in a container and, to analyse its properties, decomposes the liquid into layers with a certain finite thickness Δh, which represents a physical infinitesimal, i.e. a small but finite quantity. Leonardo highlights how for every physical infinitesimal of depth there is an increase in pressure and highlights that although the pressure increases with depth, for all the elements belonging to a given layer the pressure is the same. Another valuable example of physical infinitesimal is reported in Codex Forster II, f. 125r, where Leonardo draws a cylindrical body that can roll on a plane. Leonardo highlights that, even if the two loads are very high, as represented by the weights of 900,000 units suspended on each side, it is perfectly balanced. Then, due to the friction, a small but finite amount of additional weight, i.e. a physical infinitesimal, in this case 1/900,000 units placed on a side, can cause the transition from the static equilibrium state to a dynamic state.

3- Conclusion

In order to identify innovative methods for the current education system, which is a strong factor of social growth and humanity progress, the activity also interested the analysis of the Leonardo’s role as both learner and teacher. As above stressed, his approach to science was inherently based on experience and, in some cases, on experiments. He investigated the phenomenological world by operational procedures, by representing it through informative notes and photographic drawings, assimilable to professional reports. The analysis of the Leonardo’s notes emphasizes the priority of experience in learning sciences, the mixing of disciplinary concepts in a unified vision [3], the relation among disciplines and nature and the key role of the study method as a

prerequisite for a multi-step structured learning process. From these examples one can see that the Leonardo's approach is nowadays useful in teaching activities. His way to describe the phenomenological world allows to realize that transposition from the so called *savoir savant*, i.e. *scholarly knowledge* to *taught knowledge,* as required by Y. Chevallard.
Finally, the scientific exchanges and interactions during the fellowship period offer outstanding opportunities for productive professional relationship with regional and international research laboratories and institutions.

4- Perspectives of future collaborations with the host laboratory

The established collaborations on Leonardo's heritage have created a highly integrated team which shares several features that have contributed to the full project success and that can contribute to its future developing. Through interactions with several Centre-Val de Loire institutional members, key elements that are available to support future actions have been identified. These collaborations can count on regional, national and international partnerships and on leaders who are self-aware and mindful of the elements critical for supporting an integrated transdisciplinary approach to human and exact sciences at the center of the effort. To report an example, in planning future colloques, schools and exhibits on the subject of Leonardo, we have extensively interacted with representatives from regional, national and international institutions and from Cloux Lucè. This latter is the place where in 1516, Francis 1st hosted Leonardo da Vinci, who one can consider as a prototypical international researcher invited in Val de Loire [3], and named him *first painter, engineer and architect of the king* ; this is the place where Leonardo conceived numerous projects, brought the finishing touches to some of his masterpieces and methodically put together all of his notes, his sketches and his drawings ; this is the place where on April 23rd 1519, Leonardo considering the certainty of his death and the uncertainty of his hour, drew up his will, with the help of Guillaume Boreau, notary at the royal court; this is the place where, after having recommended his soul to God, he entrusted his co-workers; finally, this is the place where Leonardo died at the age of 67, after having received the last sacraments of the Church.
These are only some reasons for which the performed work has a great relevance for the valorisation of the Centre-Val de Loire Region.

5- Articles published in the framework of the fellowship

The knowledge transfer of the project was based not only on research activity, but also on published articles [1,2], conferences, exhibits, oral communications, and future projects.

6- Acknowledgements

First of all, the author wishes to sincerely thank Prof. Pascal Briost for his never-ending willingness, his generous friendship and his outstanding support.
The author wishes to thank *Le Studium* for the support and the affection shown over all the years of collaboration since 2009, when he met for the first time Prof. Paul Vigny, founder of Le Studium on 1996 and its President up to 2014, to whom goes a special thanking for the "thinking outside the box" discussions, for having shared his transdisciplinary vision, his "can-do" attitude, and his mentorship.
A sincere feeling of gratitude goes to the present President of *Le Studium* Mr. Yves-Michel Ginot for the friendship demonstrated over the years and for the intellectual pleasure of exchanging and delving into many subjects always looking for resonances among disciplines.
Finally, the author expresses his truly satisfaction for all the established professional and friend relationships and for the remarkable time spent in the Region.

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